Suppression of collisional shifts in a strongly interacting lattice clock

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Atomic clocks based on neutral atoms confined in optical lattices provide a unique opportunity for precise studies of quantum many-body systems. The $^{87}$Sr optical lattice clock at JILA has reached an overall fractional frequency uncertainty of $\sim 1 \times 10^{-16}$ [1,2]. This uncertainty is dominated by two contributions: atomic collisions and frequency shifts due to room–temperature blackbody radiation. The density–dependent frequency shift arises from collisions between fermionic atoms that are subject to slightly inhomogeneous optical excitation [3,4]. Several theories of the underlying frequency shift mechanism have been proposed [5,6]. A three-dimensional optical lattice clock, where each lattice site contains at most one atom, has been reported [8], and its collisional shift has been characterized with an uncertainty of $\sim 7 \times 10^{-16}$. Here we present a different and seemingly paradoxical solution to the problem: by strongly confining atoms in an array of quasi-one-dimensional potentials formed by a two-dimensional optical lattice, we dramatically increase the strength of atomic interactions. Instead of an increased collisional shift, we find that shifts are suppressed in lattice sites containing $N > 1$ atoms. Strong interactions introduce an energy gap into the system, and evolution into a many-body state in which collisions can occur is suppressed. This mechanism is akin to the continuous quantum Zeno effect, where a large dissipation rate serves to freeze a system in its initial state [9]. We demonstrate the effectiveness of this approach by reducing the density–dependent frequency shift of the JILA lattice clock to the level of $10^{-17}$, reducing the frequency shift and its uncertainty by more than a factor of ten [1]. This result demonstrates precise control of interactions in a quantum many-body system, removing an important obstacle to building optical atomic clocks with large ensembles of neutral atoms.

Strongly interacting quantum many-body systems can exhibit counterintuitive behaviors, under both equilibrium and non-equilibrium conditions. For example, in a multi-component Fermi degenerate gas, frequency shift of a microwave transition remains finite close to a Feshbach resonance [10–12]. In low dimensions, the effective strength of atomic interactions can be significantly enhanced and as a result, particles avoid each other to minimize their total energy. This tendency can lead to behavior that in many aspects resembles that of non-interacting systems. One such example is the Tonks–Girardeau regime of an ultracold Bose gas, in which the strong repulsion between particles mimics the Pauli exclusion principle, causing the bosons to behave like non-interacting fermions [13–16]. Strong dissipation in the form of large two-body losses can also simulate the Pauli exclusion principle and cause fermionization, leading to an inhibitory of particle losses in a system with fast dissipation [9,18]. Here we describe how the enhancement of atomic interactions in a strongly-interacting (but not quantum degenerate), effectively one-dimensional (1D) system suppresses collisional frequency shifts in an optical atomic clock, and we characterize this effect to an unprecedented level of precision.

Our experiment employs ultracold fermionic $^{87}$Sr atoms that are nuclear spin-polarized (e.g., $I = 9/2, m_I = +9/2$). An ultranarrow optical clock transition, whose absolute frequency has been precisely measured [19], exists between the ground $^1S_0 (|g\rangle)$ and excited metastable $^3P_0 (|e\rangle)$ states. Atoms are trapped in a deep two-dimensional (2D) optical lattice at the magic wavelength where the AC Stark shifts of $|g\rangle$ and $|e\rangle$ are matched [20]. The 2D lattice provides strong confinement along two directions ($\hat{X}$ and $\hat{Y}$), and relatively weak confinement along the remaining dimension ($\hat{Z}$).

Atoms in the lattice are sufficiently cold that they primarily occupy the ground state of the potentials along the tightly confined directions, with trap frequencies $\omega_{\perp}/2\pi \sim 90$ kHz and $\omega_{\parallel}/2\pi \sim 55$ kHz. This creates a 2D array of isolated tube-shaped potentials oriented along $\hat{Z}$, which have trap frequencies $\omega_{\parallel}/2\pi \sim 0.75$ kHz. We estimate that approximately 20% of these lattice sites are occupied by more than one atom. At a typical axial temperature $T_\perp$ of a few $\mu K$, various axial vibrational modes $n$ are populated in each tube. In a clock experiment the $|g\rangle \rightarrow |e\rangle$ transition is interrogated using Rabi spectroscopy. The atom–laser coupling is characterized by the bare Rabi frequency $\Omega_0^B$, which is defined in the absence of any motional effects. The optical frequency $\omega_L$ is detuned by an amount $\delta = \omega_L - \omega_0$ from the atomic resonance at $\omega_0$. As described in Refs. [3,21], any small projection of the probe beam along $\hat{Z}$ leads to a slightly different Rabi frequency $\Omega_n$ for each mode $\Omega_n(n\eta_Z^2)$, where $\Omega_n < \Omega_0^B$. Here $\eta_Z = k_Z a_{ho}/\sqrt{2}$ is the Lamb-Dicke parameter, $a_{ho} = \sqrt{\hbar/(m_{Sr}\omega_Z)}$ is the harmonic oscillator length, $m_{Sr}$ is atomic mass, and $k_Z$ represents a small component of the probe laser wave vector along $\hat{Z}$, resulting in a typical $\eta_Z \sim 0.05$.

To gain insight into the origin of the collisional frequency shift and the interaction-induced suppression, we consider a model system: two fermionic atoms whose electronic degrees of freedom form a two-level, pseudo-spin 1/2 system (|g\rangle and |e\rangle), confined in a 1D harmonic oscillator potential (a fully many-body treatment for an arbitrary number of atoms $N$ is discussed later in the text and developed in the Supplementary Information). The internal degrees of freedom of these two identical fermions can be expressed using a collective-spin basis, comprised of three pseudospin-symmetric triplet states.
and an antisymmetric singlet state $\langle \gamma | \rangle$. Because the atoms are initially prepared in the same symmetric internal state $\langle \gamma | \rangle$, the Pauli exclusion principle forces their spatial wave function to be antisymmetric. Thus these atoms experience no $s$–wave interactions. If the atoms are coherently driven with the same Rabi frequency ($\Omega = (\Omega_{n_1} + \Omega_{n_2})/2 = \Omega_{n_1}$), their electronic degrees of freedom remain symmetric under exchange. Consequently, these atoms will not experience any $s$–wave interactions during the excitation of the clock transition. However, if $\Delta \Omega = (|\Omega_{n_1} - \Omega_{n_2}|)/2$ is not zero, the optical excitation inhomogeneity can transfer some of the atoms to the antisymmetric spin state (singlet) that is separated from the triplet states by an energy $U$, since in this state atoms do interact. This interaction energy is what gives rise to a clock frequency shift during Rabi interrogation [6].

Figure 1 contrasts the current 2D lattice experiment with prior studies carried out in a 1D lattice [3, 4]. In a 1D lattice, the interaction energy $U$ is typically smaller than 2$\Omega$ (the energy spread of the driven triplet states). Consequently, any small excitation inhomogeneity $\Delta \Omega$ can efficiently populate the singlet state. By tightly confining atoms in a 2D lattice, one can reach the limit where $U \gg 2\Omega$. In the presence of this large energy gap, evolution into the singlet state is inhibited, and as a result the collisional frequency shift of the clock transition is suppressed. In this regime, the singlet state can only participate as a “virtual” state in second-order excitation processes and the frequency shift scales as $\Delta \Omega^2/U$. Such behavior is reminiscent of the dipolar blockade mechanism in a Rydberg atom gas [17], or the continuous quantum Zeno effect [18] where a fast dissipation rate $\Gamma$ generates a $1/\Gamma$ suppression of the decay probability, freezing the system in its initial state.

The two–particle behavior described above can be generalized to an $N$–particle system ($N > 2$). The many-body Hamiltonian can be quantitatively modeled by defining a set of effective spin operators, $S_{n_j}^{z,x}$, in the $\{e, g\}$ basis. Here the subscript $n_j$ specifies the vibrational mode. The description of the system in terms of effective spin operators is valid provided those initially populated modes remain singly occupied by either a $|g\rangle$ or an $|e\rangle$ atom during the excitation process. The latter condition is satisfied in the Lamb-Dicke regime, $\eta_{2j} \ll 1$, where one can neglect laser-induced intermode transitions. To evaluate thermally averaged quantities, we restrict the calculations to a fixed set of initially populated modes $\vec{n} = \{n_1, \ldots, n_N\}$ and sum over all possible mode configurations, weighted by the corresponding Boltzmann factors. Under the rotating wave approximation, the Hamiltonian of the system becomes (see the Supplementary Information),

$$\hat{H}_{\vec{n}}^S/h = -\delta S^z - \sum_{n=1}^N \Omega_{n_j} S_{n_j}^x - \frac{1}{2} \sum_{j=1}^N \sum_{j' \neq j} U_{n_j,n_{j'}} (\vec{S}_{n_j} \cdot \vec{S}_{n_{j'}} - 1/4).$$

$$S_{n_j}^{z,x} = \sum_{j=1}^N S_{n_j}^{z,x}$$ are collective spin operators. The quantity $U_{n_j,n_{j'}} = u I_{n_j,n_{j'}}$ measures the strength of the interactions between two atoms in the antisymmetric electronic state. The interaction parameter $u = 4\hbar a_{eg}/(m_{Sr} V)$, where $a_{eg}$ is the singlet $S = P$ scattering length. Since the volume $V = \sqrt{\frac{\hbar}{m_{Sr} \omega_X}} \sqrt{\frac{\hbar}{m_{Sr} \omega_Y}} \sqrt{\frac{\hbar}{m_{Sr} \omega_Z}}$, $u = 4\omega_\perp a_{eg}$, where $\omega_\perp = \sqrt{\omega_X \omega_Y \omega_Z}$ is the mean transverse trapping frequency. $I_{n_j,n_{j'}}$ is a mode overlap coefficient characterizing the temperature dependence. Note that $u$ is directly proportional to $\omega_\perp$ and thus increases with the transverse confinement.

The spin rotational invariance of the interaction term in $\hat{H}^S$ is key to understanding the basic physics. Due to the rotational symmetry, the interaction term is diagonal in the collective spin basis $|S, M\rangle$, $S = 0(\frac{1}{2}), \ldots, N/2$ and $|M| \leq S$. For $N = 2$, the spin basis is spanned by the triplet states $|S = 1, M = \pm 1, 0\rangle$ and the singlet $|S = 0, M = 0\rangle$. Among the collective states only the $S = N/2$ states are noninteracting. States with $S < N/2$ experience an interaction energy shift.

For a homogeneous excitation ($\Omega_{eg} = \Omega_{eg}$, where $\Omega_{eg} = \sum_{n_j} \Omega_{n_j}/N$ is the mean Rabi frequency), the Hamiltonian commutes with $S^2$ and thus $S$ is a conserved quantum number. The eigenstates of the system are just the original collective spin states up to a rotation along the $y$ axis (for $N = 2$
the eigenvalues are \(\pm \sqrt{\frac{\Omega^2_\eta + \delta^2}{2}}, 0\) for \(S = 1\) and \(U_\eta\) for \(S = 0\). If the system is prepared in the \(S = N/2\) manifold it will never experience any interaction effects, and there will be no collision-induced frequency shift. In the presence of excitation inhomogeneity, \(S\) is no longer conserved. During excitation of the clock transition, atoms will be transferred mainly between \(S = N/2\) and \(S = N/2 - 1\) states and will consequently experience a collisional frequency shift. As demonstrated in the Supplementary Information, the collisional shift experienced by atoms in multiply occupied lattice sites \((N > 2)\) remains suppressed as \(N\) increases. It also increases with \(\Delta \Omega_\eta = \sqrt{\sum_n \Omega_n^2/N - \Omega_\eta^2}\). We note that detuning inhomogeneity (for instance, atoms occupying different vibrational modes may have different resonance frequencies, \(\delta = \delta_\eta\)) has an effect similar to atom-laser coupling inhomogeneity, and the resulting shift can be modeled using Eq. A.6.

In our clock experiments, the frequency of the laser which interrogates the \(^1S_0 \rightarrow ^3P_1\) transition (the clock laser) is modulated to probe the atomic resonance at two points \(\delta_{1,2}\) on opposite sides of the line center. The mean frequency of the clock laser is steered to achieve equal population of the excited state at these two interrogation frequencies. The collisional frequency shift \(\Delta \nu\) is given by the change in the mean frequency as the particle density is varied. We determine the transition lineshape by evaluating the thermal expectation value of the excited state population as a function of detuning (calculated from Eq. A.6), and compute the shift for a given excitation fraction as \(\Delta \nu = (\delta_1 + \delta_2)/2\). A summary of these results is presented in Fig. 3 which demonstrates the suppression of the collisional frequency shift as the interaction energy is increased.

A qualitative understanding of the suppression can be developed when we calculate the frequency shift by treating \(\Delta \Omega_\eta\) as a perturbative parameter (see Supplementary Information for details). The perturbative analysis predicts, in the weak interacting regime of \(u \ll \Omega^B_0\), a shift that scales linearly with \(u\), \(\Delta \nu = A(T_\zeta, N)\eta_\eta^2 N u\). Here \(A(T_\zeta, N)\) is a temperature-dependent coefficient with contributions from \(\Delta \Omega_\eta, \Omega_\eta\), and the mode-overlap coefficient \(I_{m, n}\). We also note that \(\Delta \Omega_\eta \propto \eta_\eta^2\). This behavior is consistent with the standard mean-field expression of the density shift [10, 22, 23]. However, in the strong interacting regime of \(u \gg \Omega^B_0\), the shift is suppressed as \(\Delta \nu = B(T_\zeta, N)\eta_\eta^2 (\Omega^B_0)^2/(N u)\). Here \(B(T_\zeta, N)\) again includes the temperature-dependent effects. The suppression is consistent with the idea that in this regime the shift arises from a process that involves virtual occupations of non-finitely symmetric states.

The suppression becomes less effective if \(\Omega^B_0\) becomes comparable to \(u\), or when \(\Delta \Omega_\eta\) increases at larger temperatures. These considerations imply that clock experiments based on Ramsey interrogation will not easily satisfy the suppression conditions outlined here, since the short pulses applied in the Ramsey scheme generally have a Rabi frequency more than ten times larger than those used in Rabi spectroscopy.

To prepare the atomic system, we laser cool \(^87\)Sr atoms to about \(2 \mu\)K inside a magneto-optic trap based on the weak \(^1S_0 \rightarrow ^3P_3\) transition, and then load them into a 1D vertical lattice (along \(\hat{Y}\)) which overlaps with the MOT. The spatial distribution of occupied 1D lattice sites is determined by the vertical extent of the MOT cloud, which is approximately Gaussian with a standard deviation \(\sigma_V = 30 \mu\)m. We then adiabatically ramp up the horizontal lattice (along \(\hat{X}\)) to load the atoms into a 2D lattice. The frequencies of the two lattice beams are offset from one another by 200 MHz in order to eliminate interference effects. To remove any atoms trapped in the 1D vertical lattice outside of the 2D intersection region, we ramp the vertical lattice off and then back on again. The waist of the horizontal lattice is smaller than that of the vertical lattice by a factor of 1.3, so that no atoms remain trapped in the horizontal lattice outside of the region of overlap with the vertical lattice following the ramping procedure. The number of horizontal lattice sites occupied is then determined by the radial temperature of the vertical lattice along \(\hat{Y}\). 100 “rows” of tubes are approximately uniformly distributed along \(\hat{Y}\), while the “columns” distributed along \(\hat{X}\) are loaded according to a Gaussian distribution with standard deviation \(\sigma_H\) of 6 to 10 \(\mu\)m.

After forming the 2D lattice, we perform Doppler and sideband cooling using the \(^1S_0 \rightarrow ^3P_1, F = 11/2\) transition. Simultaneously, atoms are optically pumped to the \(m_I = +9/2\) ground state sublevel, using \(\sigma^+\)-polarized light directed along a bias magnetic field parallel to the \(\hat{Z}\) axis. We perform spectroscopy of the clock transition using a narrow linewidth laser propagating along \(\hat{Y}\). The clock laser and both lattice beams are linearly polarized along \(\hat{Z}\).

After cooling for about 30 ms, the sample temperature in the tightly-confined transverse dimension is lowered to 2 to 2.5 \(\mu\)K. We determine \(T_\zeta\) by performing Doppler spectroscopy along \(\hat{Z}\) and we vary \(T_\zeta\) between 3.8 and 7 \(\mu\)K by applying additional Doppler cooling. Trap frequencies along all three directions are determined via sideband spectroscopy and
Excited state fraction

states of parametric resonance. We quantify the number of atoms loaded into the 2D lattice by detecting fluorescence on the strong $^1S_0 \rightarrow ^1P_1$ transition at 461 nm. With a total of ~4000 atoms loaded into the 2D lattice, we estimate that 20–30% of lattice sites are multiply occupied.

We build upon the method of extracting temperature information from vibrational sideband spectra that was derived for a 1D lattice [21], and develop a model for the sideband features that we observe in a 2D lattice. These features can be understood in a fairly detailed fashion and our model reasonably describes the observed spectral features, as shown in Fig. 3. In the 1D lattice, all lattice sites are approximately equivalent, as the lattice’s Rayleigh range is much larger than the spatial extent of the MOT cloud from which the lattice is loaded; in the 2D lattice this scenario no longer holds, as lattice sites near the wings of the lattice beams’ Gaussian intensity profiles are significantly shallower than those near the center of the beam intersection region. Only after taking the distribution of site depths into account does the axial temperature extracted from the sideband model become comparable with that determined from Doppler spectroscopy along $\hat{Z}$. The spread of trap frequencies accounts for the broadened sideband lineshape without requiring an exaggerated temperature along $\hat{Z}$. The sharper features at the largest detunings from the carrier are due to atoms trapped near the center of the beam intersection region, where the trap depths are greatest.

Spectroscopy of the clock transition is performed with an 80-ms pulse, resulting in a Fourier-limited linewidth of ~10 kHz. The clock laser is locked to the atomic resonance. The high–finesse Fabry–Perot cavity [26] used to narrow the clock laser’s linewidth is sufficiently stable over short time scales that it can be used as a frequency reference in a differential measurement scheme [23]. A single experimental cycle (e.g., cooling and trapping atoms, preparing the 2D lattice, and interrogating the clock transition) requires about 1.5 s, and we modulate the sample density every two cycles. The corresponding modulation of the atomic resonance frequency relative to the cavity reference is a measurement of the density shift.

When the sample density is varied, the spatial distribution of atoms in the 2D lattice might change. As lattice sites near the center and at the edge of the beam intersection region have different trap depths, this could allow AC Stark shifts to contaminate our measurements, unless the lattice beams are tuned to the magic wavelength [20]. In the experiment we stabilize the two lattice frequencies to be $+100$ MHz and $-100$ MHz from the experimentally determined magic wavelength [11] to minimize this effect.

We perform measurements at several trap depths to directly observe the interaction-induced suppression of the collisional frequency shift. To access different interaction energies, we vary the intensity of the horizontal lattice beam.

FIG. 3. Sideband spectroscopy of atoms confined in a 2D optical lattice. The clock laser power has been increased so that the carrier is power-broadened to > 1 kHz and the sidebands are appreciably excited. The two distinct sets of sidebands reflect trap frequencies in the tightly confined $X-Y$ plane. The features at detunings of ±90 kHz are the first-order blue and red sidebands of the horizontal lattice. Due to experimental constraints, the horizontal lattice is not strictly perpendicular to $\hat{Y}$, and hence the clock laser (along $\hat{Y}$) also excites these sidebands.

FIG. 4. Experimental observation of the suppression of the collisional frequency shift with increasing interaction energy $u = 4\omega_1 \frac{a_u}{a_m}$. We varied three important parameters, $u$, $T_Z$, and $\Omega_B^R$. The symbols correspond to experimental data and the solid lines to the spin model predictions using a singlet $S-P$ scattering length, $a_{u}=-(40-50)\ a_0$, and $N=2$. The different colors and dash-styles account for different $T_Z$ and $\Omega_B$ at which experimental data were taken: Solid blue for 3.8 $\mu K$, dotted green for 4.2 $\mu K$, short-dashed yellow for 5 $\mu K$, long-dashed red for 7 $\mu K$, and dot-dashed red for 7 $\mu K$ at increased $\Omega_B^R$. The value of $u$ was varied by changing the horizontal lattice intensity, $I_Z$, which modified $\omega_Z$, $\omega_Y$, and the Lamb-Dicke parameter. The variation of $\omega_Z$ and $u$ with $I_Z$ was explicitly taken into account in theory which used $\pi/4 = 0.045$ and $\omega_Z = 2\pi \times 0.65$ kHz at the largest lattice power. Even though the value used for $\omega_Z$ is 0.85 times lower than the experimentally measured peak value, it is consistent with the spread of axial trapping frequencies across the tube array. The theoretical curves were scaled by the fraction of the atomic population in doubly occupied lattice sites. The data confirm three trends in the prediction: the collisional shift $\Delta \nu$ decreases with increasing $u$ (green triangle, yellow square, and red circle), $\Delta \nu$ increases with increasing $\Omega_B^R$ (red diamond, where $\Omega_B^R$ was increased by 2, and the interrogation time decreased by 2, yielding a constant Rabi pulse area), and $\Delta \nu$ decreases with smaller $T_Z$ (blue bow-tie).
\(I_X\), which results in the change of mainly \(\omega_X\) but also \(\omega_Z\). Since \(u \propto \sqrt{\omega_X \omega_Y \omega_Z}\), an increase of the horizontal beam power leads to a monotonic increase of \(u\). We observe a significant decrease of the collisional shift with increasing horizontal lattice power, as shown by the data points (green triangle, yellow square, and red circle) in Fig. 4. Also shown in the figure is the shift predicted by the spin model, assuming two atoms per lattice site. We scale the theoretical curves by the fraction of the atomic population in doubly occupied lattice sites. The lines of different colors represent theory results obtained at different temperatures corresponding to the actual experiment conditions under which the data were taken.

The data are consistent with the modeled shift, assuming \(a_{eg} = -(40-50) \, a_0\) (where \(a_0\) is the Bohr radius), \(\eta_Z = 0.045\), and \(\omega_L = 2\pi \times 0.65\) kHz at the largest lattice power. However, we note that \(a_{eg}\) has not been measured independently, and therefore the interaction energy cannot be accurately estimated from the known trapping frequencies.

The sign of the observed shift is negative, i.e., an increased sample density shifts the atomic resonance to lower frequencies. Previous studies of the collisional shift in a 1D optical lattice [1][3] are consistent with this observation. The simple mean-field analysis used in [21] indicated a negative scattering length, but a more sophisticated many-body treatment [6] showed that the experimental data were also consistent with a positive scattering length. From the present data set we can unambiguously conclude that \(a_{eg}\) is negative.

We have also studied the dependence of the collisional shift on the Rabi frequency used to drive the clock transition. \(\Omega_\theta^B\) was increased by a factor of two, and the interrogation time was decreased by 2, yielding a constant Rabi pulse area. Under these conditions, we observe that the collisional shift increases sharply, to \((4.1 \pm 0.6) \times 10^{-16}\) (red diamond in Fig. 4), confirming that the shift suppression mechanism will not operate effectively for short, higher Rabi frequency pulses.

We have made an extensive series of collisional shift measurements at the largest trap depths available to us. The results of these measurements are displayed in Fig. 5. The free-running clock laser has a stability of about \(5 \times 10^{-16}\) at time scales of 1–10 seconds [26]. Therefore, a substantial integration time is required to determine the collisional shift with an uncertainty of \(1 \times 10^{-17}\). Frequency drifts are minimized by measuring the long-term drift in the resonance frequency (relative to the ultrastable reference cavity) and applying a feed-forward correction to the clock laser. The correlation between the atomic resonance frequencies and the density of trapped atoms was calculated by analyzing groups of three consecutive measurements and eliminating any residual frequency drift [27]. Approximately 60 hours of data were acquired at \(T_Z = 7\) \(\mu\)K over a \(\sim 2\) month time period for the record shown in Fig. 5(a). A histogram of the entire data set demonstrates that measurement errors were consistent with uncorrelated white noise (Fig. 5(b)). From the weighted mean and weighted standard error of the data set, we determine the collisional shift of our 2D optical lattice clock to be \((5.5 \pm 1.4) \times 10^{-17}\). At a lower \(T_Z\) of 3.8 \(\mu\)K, the collisional shift is reduced to \((1 \pm 6) \times 10^{-17}\) (blue bow-tie in Fig. 4 and the record in Fig. 5(c) and (d)).

Figures 4 and 5 demonstrate the suppression of the clock frequency shift as the system approaches the strongly interacting regime. We note that, relative to previous measurements of collisional shifts in a 1D optical lattice [1][3], the atomic density in a 2D lattice is much higher. After accounting for the increased temperature in our current experiments, we find that the local density in a lattice site occupied by two particles is \(\sim 1.4 \times 10^{13}\) \(cm^{-3}\), an order of magnitude larger than the average density in earlier 1D lattice experiments. Therefore, given a similar level of excitation inhomogeneity, if the collisional shift in the 2D lattice were not suppressed, we would expect a significantly larger shift than that in the 1D lattice experiments, even after assuming that only \(20 - 30\%\) of lattice sites are contributing.

The results presented here demonstrate that detailed understanding of a many-body quantum system can result in dramatic improvements in the areas of precision measurement and atomic clocks. This advance removes an important obstacle to further increasing the precision and accuracy of neutral atom-based optical clocks. Increasing the number of atoms loaded into our 2D lattice system will enable us to improve the stability of our clock without imposing an onerous systematic effect. As clock lasers become more stable, we will be able to increase the duration of the Rabi interrogation pulse, thus decreasing the Rabi frequency. This will allow optical lattice clocks to operate in the regime where the density shift is fully suppressed \(\sim 1/\mu\) and further reduce the collisional shift systematically well into the \(10^{-18}\) domain. This, together with the fact that in the strongly-interacting regime the collisional shift will remain suppressed as more atoms are loaded into individual lattice sites, will enable neutral atom clocks to operate with the large sample sizes needed to achieve the highest possible stability. This approach also sidesteps systematic issues that might affect a 3D optical lattice clock, which arise from site-dependent ellipticity of the lattice polarization and the associated vector and tensor shifts of the clock transition.

FIG. 5. Data records of collision-induced frequency shift measurements for $^{87}$Sr atoms confined in a 2D optical lattice. For the records displayed in (a) and (c), each point represent a data set collected from a continuous operation of the Sr clock. The error bars represent the standard error for that set of data. (b) and (d) show the corresponding histograms of the frequency shift records displayed in (a) and (c), respectively. Under typical clock operating conditions ($N \simeq 2000$), the weighted mean and the weighted standard error of the fractional frequency shift are $(5.5 \pm 1.4) \times 10^{-17}$ at $T_Z = 7 \mu K$ ((a) and (b)) and $(1 \pm 6) \times 10^{-17}$ at $T_Z = 3.8 \mu K$ ((c) and (d)).


[25] Boyd, M. M. *et al.* $^{87}$Sr lattice clock with inaccuracy below 1

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Author contributions M.D.S, M.B., Y.L., S.B., M.J.M. and J.Y. designed and performed the experiment. A.M.R. developed the theory. All contributed to the writing of the manuscript.
Appendix: Supporting online material

1. Many-body Hamiltonian for spin polarized fermionic atoms

Here we will consider a spin polarized ensemble of fermionic atoms (e.g. $I = 9/2$) with two accessible electronic degrees of freedom associated to the $^1S_0(g) - ^3P_0(e)$ states. We focus on the case where the atoms are trapped in an external potential $V(R)$ that is the same for $g$ and $e$ (i.e. at the “magic wavelength” [1]). If the atoms are illuminated by a linearly polarized laser beam with bare Rabi frequency $\Omega_0^B$ they are governed by the following many-body Hamiltonian [2–5]

\[
\hat{H} = \sum_\alpha \int d^3R \hat{\Psi}_\alpha^\dagger \left( -\frac{\hbar^2}{2m_{Sr}} \nabla^2 + V(R) \right) \hat{\Psi}_\alpha
\]

\[+ \hbar u_{eg} \int d^3R \hat{\rho}_g \hat{\rho}_g + \hbar \omega_H \int d^3R (\hat{\rho}_e - \hat{\rho}_g)
\]

\[- \frac{\hbar \Omega_0^B}{2} \int d^3R (\hat{\Psi}_e^\dagger e^{-i(\omega_L t - k \cdot R)} \hat{\Psi}_g + h.c.). \quad (A.1)
\]

Here $\hat{\Psi}_\alpha(R)$ is a fermionic field operator at position $R$ for atoms with mass $m_{Sr}$ in electronic state $\alpha = g (^1S_0)$ or $e (^3P_0)$, while $\hat{\rho}_\alpha(R) = \hat{\Psi}^\dagger_\alpha(R) \hat{\Psi}_\alpha(R)$ is the corresponding density operator. Since polarized fermions are in a symmetric nuclear state, their $s$-wave interactions are characterized by only one scattering length $a_{eg}$, with the corresponding interaction parameter $u_{eg} = 4\pi\hbar a_{eg}^3/m_{Sr}$, describing collisions between two atoms in the antisymmetric electronic state. The laser with frequency $\omega_L$ and wavevector $k$ is detuned from the atom transition frequency $\omega_0$ by $\delta = \omega_L - \omega_0$.

We consider the situation in which a deep 2D lattice freezes the atomic motion in the transverse $X - Y$ plane creating an array of one dimensional tubes. Along the longitudinal $Z$-direction the net effect of the lattice is to induce a weak harmonic confinement with frequency $\omega_Z$. While the deep 2D lattice confines the atoms to the lowest vibrational mode, we allow mobility along the longitudinal direction. It is then convenient to expand the field operator, $\hat{\Psi}_\alpha(R)$, in a harmonic oscillator basis, $\hat{\Psi}_\alpha(R) = \phi_0^\alpha(X) \phi_1^\alpha(Y) \sum_n \hat{c}_{n\alpha} \phi_n(Z)$, where $\phi_0^X$ and $\phi_0^e$ are, respectively, the transverse and the longitudinal harmonic oscillator eigenmodes and $\hat{c}_{n\alpha}$ creates a fermion in mode $n$ and electronic level $\alpha$.

Following Refs. [4, 6,8], we assume that the probe is slightly misaligned with a small component along the $z$-direction: $k = k_Y Y + k_Z Z$ with $|k_Z/k_Y| \ll 1$. Defining $\Omega_{n,n'} = \Omega_0 e^{-\left(n \pi^2\right)/2L_0^2(\eta^2)}(\phi_n(Z)) \phi_{n'}(Z)$ where $\eta_{Y,Z} = k_Y Z a_{ho}/\sqrt{2} \ll 1$ are the Lamb-Dicke parameters and $L_n$ are Laguerre polynomials [9], laser induced sideband transitions can be neglected if $\Omega_{n,n'^\perp} \ll \omega_Z$. In this regime, $\hat{H}$ can be rewritten in the rotating frame as

\[
\hat{H} = -\hbar \delta \sum_{n=0}^\infty \hat{n}_{en} + \sum_{\alpha,n=0}^\infty E_n \hat{n}_{\alpha n} - \hbar \Omega_0^B \sum_{n=0}^\infty \left( \hat{c}_{en}^\dagger \hat{c}_{en} + h.c. \right) + \hbar \frac{u}{2} \sum_{n=0}^\infty \sum_{n'=0}^\infty \sum_{n''=0}^\infty \sum_{n'''=0}^\infty I_{nn'n''n'''} \hat{c}_{en}^\dagger \hat{c}_{en} \hat{c}_{gn}^\dagger \hat{c}_{gn}^\dagger, \quad (A.2)
\]

where

\[
u = 4\omega_\perp a_{eg}^2 \quad (A.3)
\]

and

\[
I_{nn'n''n'''} = \frac{\int e^{-2\xi^2} H_n(\xi) H_{n'}(\xi) H_{n''}(\xi) H_{n'''}(\xi) d\xi}{\sqrt{2^{n+n'+n''+n'''}(n!)^4(n'!)^4(n''!)^4(n'''!)^4}}. \quad (A.4)
\]

Hamiltonian becomes

\[
\hat{H}_S/R = -\frac{\delta S^z S^z}{\hbar} - \sum_{n=1}^{N} \Omega_{n,j} S_n^e - \sum_{j=1}^{N} \sum_{j'=j}^{N} \frac{U_{n,j,n',j'}}{2} (S_{nj} \cdot S_{nj'} - 1/4). \quad (A.5)
\]

Here $S_n^j = \frac{1}{2} \sum_{\alpha,\alpha'} \hat{c}_{\alpha n}^\dagger \sigma_{\alpha \alpha'} \hat{c}_{\alpha' n}$, where $\sigma$ are Pauli matrices in the $\{g, e\}$ basis, $S^z = g - e = \sum_{n=1}^{N} S_n^z$, and constant terms were dropped. The quantity $U_{n,j,n',j'} = u I_{n,j,n',j'} = a I_{n,j,n',j'}$.

The rotational invariance of the interaction term in $\hat{H}_S/R$ is key for the understanding of the basic features of the model. Due to the rotational symmetry the interaction term is diagonal in the collective angular momentum basis $|S,M,q\rangle$, satisfying $S^2 |S,M,q\rangle = S(S+1) |S,M,q\rangle$ and $S^z |S,M,q\rangle = M |S,M,q\rangle$, with $S = 0 \left( \frac{1}{2} \right), \ldots, N/2$ and $-S \leq M \leq S$. Here the extra label $q$ is required to uniquely specify each state. The fully symmetric (Dicke) $S = N/2$ states do not interact. They are unique and the label $q$ can be
omitted for them.

To proceed further, we use the fact that \( U_{n_j, n_{j'}} \) is a slowly varying function of \( |n_j - n_{j'}| \), \( I_{n_j, 0, n_{j'}, 0} \to \frac{1}{\pi \sqrt{2|n_j - n_{j'}|}} \).

In the regime ( \( k_B T \gtrsim N \hbar \omega_2 \) ) where the occupied modes \( \tilde{n} \) are sufficiently sparse for the behavior of \( U_{n_j, n_{j'}} \) to be dominated by its slowly varying part, we can approximate \( U_{n_j, n_{j'}} \to U_\tilde{n} \equiv \sum_{j, j' \neq j} U_{n_j, n_{j'}/(N(N - 1))} \) and

\[
\hat{H}_\tilde{n}^S/\hbar \approx -\delta \hat{S}^z - \sum_{j=1}^N \Omega_{n_j} (\cos \theta \hat{s}^x_{n_j} + \sin \theta \hat{s}^y_{n_j}) - \frac{U_\tilde{n}}{2} (\tilde{\hat{s}} \cdot \tilde{\hat{s}}). \tag{A.6}
\]

Again constant terms have been dropped.

Under this approximation it is possible to have an analytic treatment of the many-body dynamics. With this purpose in mind, it is convenient to go to a rotated basis and rewrite the Hamiltonian as

\[
\hat{H}_\tilde{n}^S/\hbar = \sqrt{\delta^2 + \Omega_\tilde{n}^2} \hat{s}^z + \sum_{j=1}^N \delta \Omega_{n_j} (\cos \theta \hat{s}^x_{n_j} + \sin \theta \hat{s}^y_{n_j}) - \frac{U_\tilde{n}}{2} (\tilde{\hat{s}} \cdot \tilde{\hat{s}}). \tag{A.7}
\]

where \( \hat{s}^z = \hat{\vec{s}} \cdot \hat{\vec{s}}, \hat{\vec{s}} = (\sin \theta, 0, \cos \theta) \), \( \theta = \arcsin \left( \frac{\Omega_{\tilde{n}}}{\sqrt{\delta^2 + \Omega_{\tilde{n}}^2}} \right) \)

\( \delta \Omega_{n_j} = \Omega_{n_j} - \Omega_{\tilde{n}} \) and \( \Omega_{\tilde{n}} = \frac{1}{N} \sum_{j=0}^N \Omega_{n_j} \) the mean Rabi frequency.

We will consider \( \delta \Omega_{n_j} \) as our perturbative parameter. To zero order in it, the eigenstates of the Hamiltonian are conveniently described in terms of angular momentum eigenstates in the rotated basis, \( |S, m, k \rangle \) (the quantum number \( S \) is conserved in rotations), satisfying

\[
\hat{s}^z |S, m, k \rangle = S(S + 1) |S, m, k \rangle \quad \text{and} \quad s^z = m |S, m, k \rangle.
\]

At time \( t = 0 \) all the atoms are in the g state and thus initially \( S = N/2 \). Non-zero \( \{ \delta \Omega_{n_j} \} \) induce transitions outside the \( S = N/2 \) manifold. However, to first order in perturbation theory, the term proportional to \( \{ \delta \Omega_{n_j} \} \) can only induce transitions to states with \( S = N/2 - 1 \) due to its linear dependence on \( \delta \hat{s} \cdot \hat{\vec{s}} \). This implies that the knowledge of the eigenstates and eigenvalues within the \( S = N/2, N/2 - 1 \) manifolds are enough to characterize the perturbative dynamics.

The \( |N/2, m \rangle \) eigenstates are just the well known Dicke states invariant under particle permutation. They have energies given by \( E_{N/2, m} = \hbar \omega_{N/2, m} = \hbar \sqrt{\delta^2 + \Omega_{\tilde{n}}^2 m} \). Since the initially prepared state is a fully polarized state in the old basis, in the rotated basis it corresponds to a superposition of \( |N/2, m \rangle \) states with amplitude probabilities determined by the Wigner rotation matrices:

\[
|\psi(0)\rangle = |gg \ldots g \rangle = \sum_{m=-N/2}^{N/2} \sqrt{\frac{N}{m + N/2}} |N/2, m \rangle (\Pi/2) \sin^{N/2+m} (\theta/2) |N/2, -m \rangle \tag{A.8}
\]

The states with \( S = N/2 - 1 \) are the so called spin-wave states. They can be written in terms of Dicke states as:

\[
|N/2 - 1, m, k \rangle = \left( \frac{(N - 1)}{(N/2 - m + 1)(N/2 - m)} \right)^{1/2} \times \sum_{n=1}^N e^{i2\pi kn/N} s_n^+ |N/2, m - 1 \rangle \tag{A.9}
\]

with \( k = 1, \ldots N - 1 \). These states have energy \( E_{N/2 - 1, m}/\hbar = \omega_{N/2 - 1, m} = \sqrt{\delta^2 + \Omega_{\tilde{n}}^2 m} + \frac{N}{2} U_{\tilde{n}} \). From the energy it is clear that the population of these states will give raise to an interaction energy shift. For the simple case \( N = 2 \) described in the main text, there is a unique spin wave state which correspond to the singlet state which has energy \( U_{\tilde{n}} \).

If we write our time evolving many-body state as

\[
|\psi(t)\rangle = \sum_{m} c_m(t) e^{-i\omega_{N/2, m} t} |N/2, m \rangle + \sum_{m, k} b_{m, k}(t) e^{-i\omega_{N/2 - 1, m} t} |N/2 - 1, m, k \rangle \tag{A.10}
\]

then the excite state population is given by

\[
N_{\tilde{n}} (t) = \frac{N}{2} + \langle S_z(t) \rangle \tag{A.11}
\]

\[
= \frac{N}{2} + \cos \theta \langle s_z(t) \rangle - \sin \theta \langle s_z(t) \rangle \tag{A.12}
\]

The following transition matrix elements are required for the perturbative calculations:
\[\langle N/2, m | 2s_n^2 | N/2 - 1, \tilde{m}, k \rangle = 2e^{2i\pi kn/N} \sqrt{\frac{(N/2)^2 - m^2}{N^2(N-1)}} \delta_{m, \tilde{m}} \] (A.13)

\[\langle N/2, m | s_n^+ | N/2 - 1, \tilde{m}, k \rangle = -e^{2i\pi kn/N} \sqrt{\frac{(N/2 + m)(N/2 + m - 1)}{N^2(N-1)}} \delta_{m, \tilde{m}+1} \] (A.14)

\[\langle N/2, m | s_n^- | N/2 - 1, \tilde{m}, k \rangle = -e^{2i\pi kn/N} \sqrt{\frac{(N/2 - m)(N/2 - m - 1)}{N^2(N-1)}} \delta_{m, \tilde{m}-1} \] (A.15)

\[\langle N/2 - 1, m, k | 2s_n^2 | N/2 - 1, \tilde{m}, k \rangle = (-2e^{2i\pi (k-k)n/N} + N\delta_{k,\tilde{k}}) \frac{2m}{N(N-2)} \delta_{m, \tilde{m}} \] (A.16)

\[\langle N/2 - 1, m, k | s_n^+ | N/2 - 1, \tilde{m}, k \rangle = (-2e^{2i\pi (k-k)n/N} + N\delta_{k,\tilde{k}}) \times \sqrt{\frac{(N/2 + m - 1)(N/2 - m)}{N(N-2)}} \delta_{m, \tilde{m}+1} \] (A.17)

\[\langle N/2 - 1, m, k | s_n^- | N/2 - 1, \tilde{m}, k \rangle = (-2e^{2i\pi (k-k)n/N} + N\delta_{k,\tilde{k}}) \times \sqrt{\frac{(N/2 + m)(N/2 - m - 1)}{N(N-2)}} \delta_{m, \tilde{m}-1} \] (A.18)

The matrix elements in Eqs. (A.16) and (A.18) are valid only for \( N > 2 \) and are all zero for \( N = 2 \).

Using those matrix elements one can show after some algebra that \( N_{\tilde{r}}^e(t) \) depends on \( \Omega_n \) only through the mean Rabi frequency, \( \bar{\Omega}_n \), and the root-mean-square Rabi frequency, \( \Delta \Omega_{\tilde{r}} = \sqrt{\sum_n \Omega_{\tilde{r}}^2/N - \bar{\Omega}_n^2} \). More explicitly

\[N_{\tilde{r}}^e(t) = N_{\tilde{r}}^{e(0)}(t) + \Delta \Omega_{\tilde{r}}^2 N_{\tilde{r}}^{e(2)}(t) + O(\Delta \Omega_{\tilde{r}}^4)\] (A.19)

where the superscript \(^{(0)}\) indicates a homogeneous excitation.

\[N_{\tilde{r}}^{e(0)}(t) = N - \frac{\Omega_{\tilde{r}}^2}{\bar{\Omega}_n^2 + \delta^2} \sin^2\left(\frac{t \sqrt{\Omega_{\tilde{r}}^2 + \delta^2}}{2}\right),\] (A.20)

Note that since \( \langle N/2, m | \hat{s}_{x,y,z} | N/2 - 1, \tilde{m}, k \rangle = 0 \) then \( N_{\tilde{r}}^{e}(t) \) does not have first order corrections in \( \Delta \Omega_{\tilde{r}} \).

### 2. Analytic evaluation of the clock frequency shift (CFS)

In clock experiments based on Rabi interrogation the clock frequency shift, CFS, \( \Delta \nu \) is measured by first locking the spectroscopy laser at two points, \( \delta_{1,2} \), of equal height in the transition lineshape (equal final excited state fraction under the initial condition of all atoms in state \( g \)) and then determining the change in the mean frequency as the interaction parameters or density are varied, \( \Delta \nu = (\delta_1 + \delta_2)/2 \). To calculate the shift, we Taylor expand \( \delta_{1,2} \) around the zero order values, \( \pm \delta_{1}^{(0)} \), and to lowest nonvanishing order obtain

\[\Delta \nu_{\tilde{r}} = \frac{N_{\tilde{r}}^{e(2)}(t, \delta_1^{(0)}) - N_{\tilde{r}}^{e(2)}(t, -\delta_1^{(0)})}{2 \frac{\partial N_{\tilde{r}}^{e(2)}(t, \delta)}{\partial \delta} |_{\delta_1^{(0)}}} (A.21)\]

\[= \frac{C_{\tilde{r}}}{4\pi D_{\tilde{r}}} \] (A.22)

with
\[ D = \frac{\delta \Omega^2 \left(-2 + 2 \cos \left[t \sqrt{\delta^2 + \Omega^2} \right] + t \sqrt{\delta^2 + \Omega^2} \sin \left[t \sqrt{\delta^2 + \Omega^2} \right] \right)}{\left(\delta^2 + \Omega^2\right)^2} \]  

\[ C = \frac{2\Delta \Omega^2 \delta}{N^2 U^2 \left(\delta^2 + \Omega^2\right)^3 \left(-N^2 U^2/4 + \delta^2 + \Omega^2\right)^2} \]
\[
\times \left(2NU\Omega^4 \left(\delta^2 + \Omega^2\right)^2 - 2NU \cos[tNU/2]\Omega^2 \left(-NU/2 + \Omega\right) \left(NU/2 + \Omega\right) \left(\delta^2 + \Omega^2\right)^2 \right) \\
+ N^3 U^5 \Omega^2 / 32 \left(-7\delta^2 + 2\Omega^2\right) + N^3 U^3 / 8 \cos \left[t \sqrt{\delta^2 + \Omega^2} \right] \Omega^2 \left(-N^2 U^2/4 + \delta^2 + \Omega^2\right) \left(\delta^2 + 2\Omega^2\right) \\
- N^3 U^3 / 8 \left(8\delta^2 + 5\delta^4 \Omega^2 + 3\delta^2 \Omega^4 + 6\Omega^6\right) \\
+ 2NU \cos \left[t \sqrt{\delta^2 + \Omega^2} \right] \left(-\cos[tNU/2] \left(-NU/2 + \Omega\right) \left(NU/2 + \Omega\right) \left(\delta^2 + \Omega^2\right)^2 \left(2\delta^2 + \Omega^2\right) \right) \\
+ \Omega^2 \left(N^4 U^4 \delta^2 / 32 + \left(\delta^2 + \Omega^2\right)^2 \left(2\delta^2 + \Omega^2\right) - N^2 U^2/4 \left(\delta^2 + \Omega^2\right) \left(5\delta^2 + \Omega^2\right) \right) \\
+ \sin \left[t \sqrt{\delta^2 + \Omega^2} \right] \left(-tNU \Omega^2 \sqrt{\delta^2 + \Omega^2} \left(-N^2 U^2/4 + \delta^2 + \Omega^2\right) \left(N^2 U^2/4 \left(\delta^2 - 2\Omega^2\right) \right) \right) \\
+ 2\Omega^2 \left(\delta^2 + \Omega^2\right) + 4 \left(\delta^2 + \Omega^2\right) \left(N^4 U^4 / 16 + \Omega^4 + N^2 U^2/4 \left(\delta^2 - 2\Omega^2\right) \right) \sin[tNU/2] \\
- N^3 U^3 / 8 \Omega^2 \left(-N^2 U^2/4 + \delta^2 + \Omega^2\right) \left(\delta^2 + 2\Omega^2\right) \sin \left[t \sqrt{\delta^2 + \Omega^2} \right] \right). \]  

(A.24)

Here we have omitted the subscript \( \vec{n} \) but it is understood.

So far we have assumed a fixed set of populated modes, \( \vec{n} \). At finite temperature, expectation values need to be calculated by averaging over all possible combinations of modes \( \{ \vec{n} \} \) weighted according to their Boltzmann factor:

\[ \langle \mathcal{O} \rangle_{T \bar{T}} = \frac{\sum_{\vec{n}} \mathcal{O}_{\vec{n}} \exp \left[-E_{\vec{n}} / (k_B T)\right]}{\sum_{\vec{n}} \exp \left[-E_{\vec{n}} / (k_B T)\right]}, \]  

(A.25)

with \( E_{\vec{n}} = \sum_j E_{n_j} \). The thermal averaged expression of the shift becomes

\[ \Delta \nu \equiv \langle \Delta \nu_{\vec{n}} \rangle_{T \bar{T}} = \frac{(C_{\vec{n}})_{T \bar{T}}}{4\pi(D_{\vec{n}})_{T \bar{T}}} \]  

(A.26)

Grouping all temperature dependent terms in a temperature dependent coefficient, we obtain the following scaling behavior of the shift: \( \Delta \nu \propto A(T_Z, N) \eta_1^2 Nu \) in the weakly interacting regime consistent with prior mean field analysis [10, 11] and \( \Delta \nu \propto B(T_Z, N) (\Omega_B^0)^2 \eta_1^2 / (Nu) \) in the strongly interacting regime.